

Strategies to improve problem-solving skills

Irene Stone

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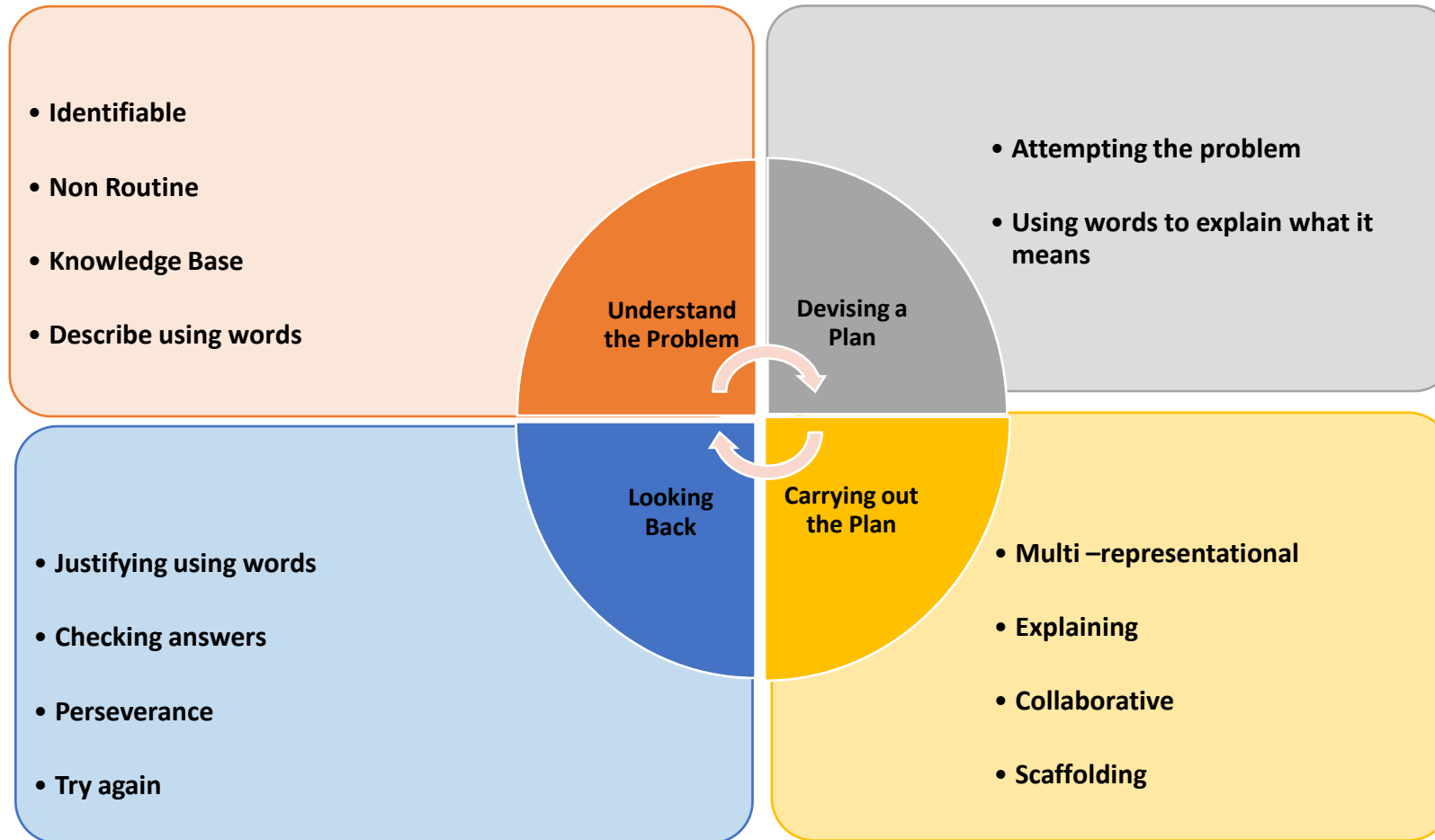
Two strategies:

- How working collaboratively through an online environment can support problem-solving skills of students
- Weekly puzzle competition – whole school

What is problem solving?

- *“Problems are situations with no obvious solution, and solving problems requires thinking and learning in action”* (OECD, 2014, p. 26).
- Problem solving is *“concerned with non-routine tasks”* where one *“can’t apply previously learned procedures”* (OECD, 2014, p. 30). The chief examiner reports recommend that teachers engage students in non-routine tasks (SEC, 2015a, 2015b).

Polya's 4 step Problem Solving process



(Polya, 1957, p.xvii)

Theory – themes that emerged from literature

- **Reflecting and trying again** are key elements of problem solving (DES, 2013; Hurme & Järvelä, 2005; Kim & Hannafin, 2011)
- When problem solving, one should **use words** to describe, explain, or justify what they're doing (Hurme & Jarvela, 2005; Muir et al., 2008; SEC, 2015a).
- Students should aim for a **multi-representational** approach when problem solving (Boaler, 2010; Schoenfeld, 1992).
- Problem solving should be **collaborative** (Kim & Hannafin, 2011; Tanner & Jones, 2000).
- **Scaffolding** is where teachers do not tell students too much but instead guide them by asking stimulating questions; scaffolding can enhance problem-solving skills (Polya, 1957, p. v; Tan Yeen-Ju, Mai, & Selvaretnam, 2015, p. 845; Tanner & Jones, 2000, p. 29).

Background

- Problem Solving (PS) - an integral part of mathematical learning (Boaler, 2010; Polya, 1957).
- Problem-solving skills in demand in today's workplace and in education (OECD, 2014, p. 26).
- Project Maths course was introduced in 2008. One of the aims of the new syllabus was to embed problem-solving skills amongst students; to steer them away from the procedural approach which dominated the “traditional” mathematics classroom (NCCA, 2005, p. 18)
- A need for **collaboration** and **digital skills**, amongst others, to be embedded in the new Junior Cycle (DES, 2015a, p. 7)
- One of the key points of the Digital Strategy for Schools is to embed digital skills within all subjects of the curriculum (DES, 2015b).
- The Minister of Education in 2015, Jan O' Sullivan, encouraged all teachers to use technology “to give learners the tools to collaborate and to examine engaging problems”.

(<http://www.education.ie/en/Press-Events/Press-Releases/2015-Press-Releases/PR15-10-07A.html>)

The problem

- Since the introduction of Project Maths, reports suggest that students still lacking in PS skills, in particular in problems involving algebra – Chief Examiner Reports, PISA 2012 and PISA 2015 results (Perkins & Shiel, 2014, p. 7; SEC, 2015a, 2015b).
- Latest, PISA 2015 - students are still struggling with “higher-order” skills such as problem solving (DES, 2016, p.11).
- Research suggests that students’ exposure to problem-solving skills in a *classroom* environment is limited (Bray & Tangney, 2016; Kim & Hannafin, 2011; Schoenfeld, 1992).
- Teachers feel pressure of terminal exam and are therefore reluctant to facilitate methodologies such as group-work and setting open-ended tasks (NCCA, 2012, p. 11)

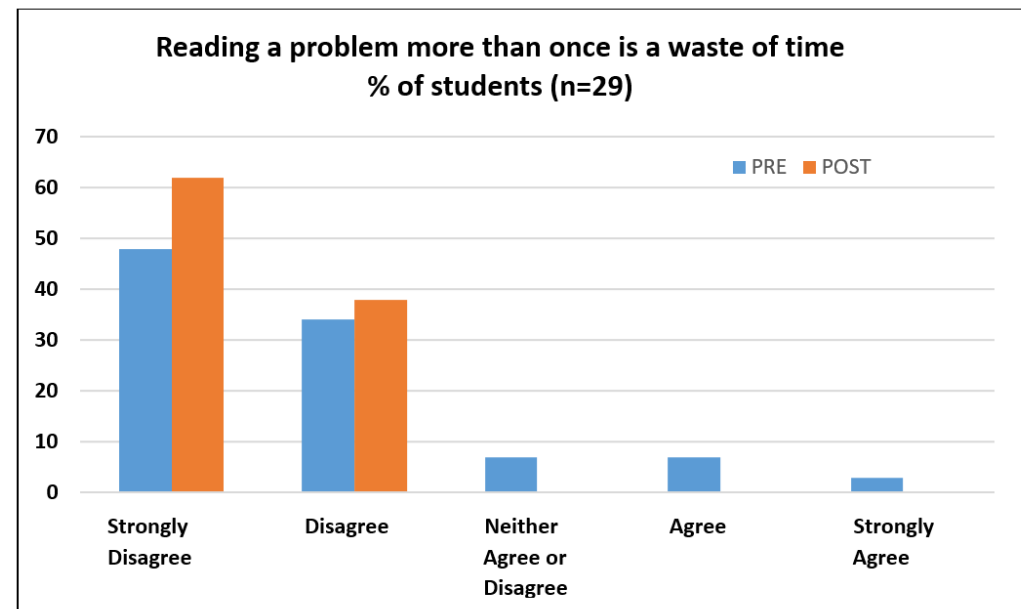
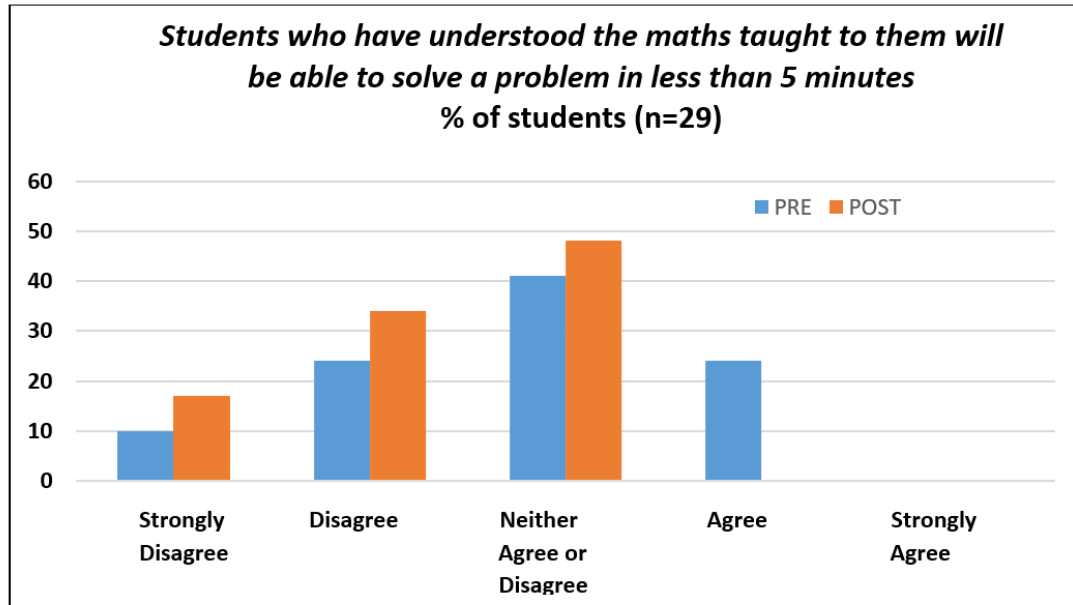
The research

- This research investigated if working collaboratively through an online environment can support problem-solving skills.
- Google Docs, the online environment used for this research, allows for collaborative editing of a document.
- An exploratory case study was carried out which involved 30 post-primary students from a 2nd year Mathematics class, ages 13-14 years. Working in groups of 4 or 5 they were given problems to solve on the Google Doc environment over a two week period.
- The research employed mixed methodologies.
- Findings of the research concluded that working collaboratively through an online environment can support problem-solving skills of post-primary mathematics students.

Findings demonstrated that in the Google Doc environment...

- Students were **reflecting** and/or **trying again**.
- They were **using words** to describe what they were doing
- They applied a **multi-representational** approach to problem solve
- The Google Doc environment supported **scaffolding** by the teacher and others in the group.
- They were working **collaboratively**.

Reflecting and/or trying again



Evidence of a willingness to accept that solving a problem involves work and time; that it is important to reflect on a problem and persist with it - displaying a productive disposition

The difference between the blues always go up in twos. The first amount of blues are 8, then 10, so on and so forth. So our formula is.. (me and ? solved this together)

$$\underline{2x + 6}$$


x = the pattern we are looking for, in this case, 5. So we multiplied it by two because it goes up in twos each time. And then we plus 6 because that's where it starts after you also take away the two blue that go with the yellow.

Check:

Let's do it with 5	$2(5) + 6$ $10 + 6 = 16$
Let's do it with 100	$2(100) + 6$ $200 + 6 = 206$
Let's do it with 200	$2(200) + 6 = 406$

**Students checking
their formula**

**Teacher encouraged
students to develop
a strategy for
checking their
answer (SEC, 2015a)**



Irene Stone
19:50 9 Dec
Resolve

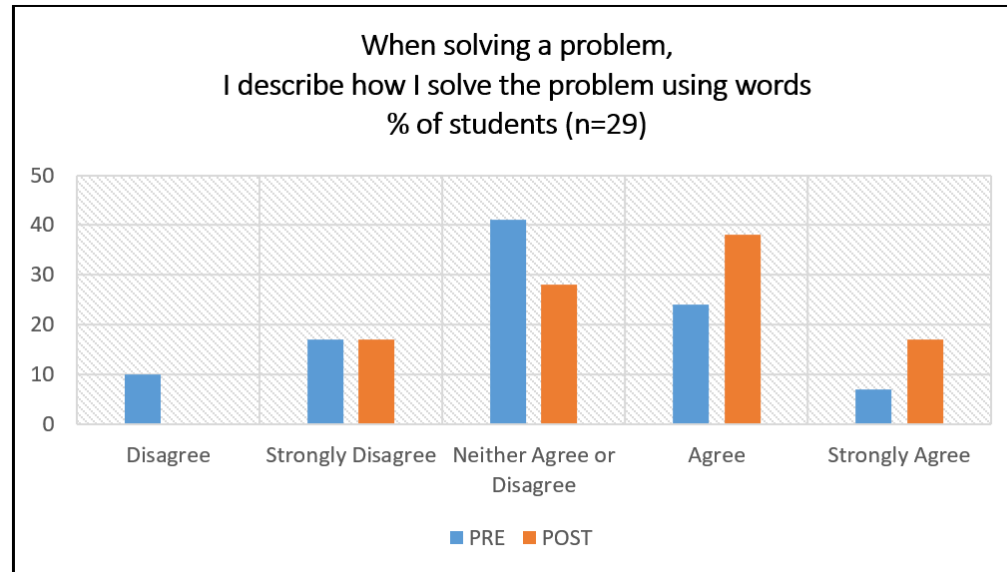
fantastic [redacted] and [redacted] When you make a formula it is always worth checking it for the patterns you have in front of you. i.e. put in $x=1$: you get $2(1)+6=8$ which is correct. There are 8 blue tiles in the first diagram! That way you know your formula is correct. Keep up the great work.

- Students were conscious that the others in their group would be looking at their work; this motivating factor causes them to reflect more through explaining the way they solved a problem.
- The comments tool allowed the teacher to remind students to check.

“On the google doc ... I always wrote out a lot of information on why I was doing stuff”

“I was aware unlike in my copy that people were gonna be looking at it... it's just as in your copy it's just you were as in the google doc its more people so you have to explain it to everyone so they know what you are doing .”

Using words



It says what is the maximum area he can have with 20m fencing.

First I found out what 4 numbers add up to twenty since a rectangle has 4 sides:

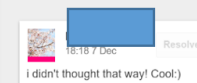
$$5+5+5+5=20$$

$$9+9+1+1=20$$

$$6+6+4+4=20\text{.....ETC}$$

It said area and to get area you have to multiply length by width.

So I figured out what length x width equaled the highest number and so I got $5 \times 5 = 25$ So I think the highest possible area would be 25m squared. Although you may consider it a square but we learned that a square has all the the qualifications to be a rectangle: it has 4 sides and corners plus all right angles so therefore a square is a rectangle but a rectangle is not a square.



So me and s done this the other day on her group so i think i know how to do it. I'll show you what we did.

So already we can see that each time the yellows add one and the blues add two. So if we were doing a formula we would know that no matter what whatever one we're looking for will always be the same amount of yellows. For the first one there's eight blues so if i'm looking for no.5 i would do $2(x) + 8$ because 2 is the difference in each one. So wait lets go back to yellow. The formula for that would be $x(1)$ (x being whatever pattern we're looking for) nothing because the first one is just one so for blue its the same thing so 8 is like the starting amont except 8 isnt the starting amount because for each yellow theres two blue so for the first one you take away two so the formula for the blue is $2x + 6$ does this make any sense?

I gatchu

- It was because other people were viewing their work, students felt the need to explain what they were doing.

“You’re not going to write in your copy you are just write examples in your copy”

“I think that using words helped to describe what I was doing and how other people can understood the problem better”

A student came up with a formula. Another student asked her how she got the formula. This encouraged the first student to explain in words what she did.

formula:

$$\frac{x(x+1)}{2}$$

can you explain how you got the formula?

Q3. The 100th Staircase:

$$\frac{100(100+1)}{2} = \frac{100 \times 101}{2} = \frac{10,100}{2} = 5,050$$

Q1. This is a quadratic pattern because the 2nd changes are constant. They are all going up in 1. It's not a linear pattern because the 1st changes are not constant.

g this is amazing! How did you figure this out??

example :

$$\frac{x(x+1)}{2} = \frac{2(2+1)}{2} : \frac{2(3)}{2} = \frac{6}{2} = 3$$

At the start we add 2(pattern) + 1(change) . We get 3 so then we multiply 2(pattern) = 6. We put 6 over 2 getting = $\frac{6}{2}$, so then we divide 6 by 2 and we get the answer 3.

Multi-representational

Problem: One side of a rectangle is 4m bigger than the other side. The area of the rectangle is 60metres squared. What is the perimeter of the rectangle?

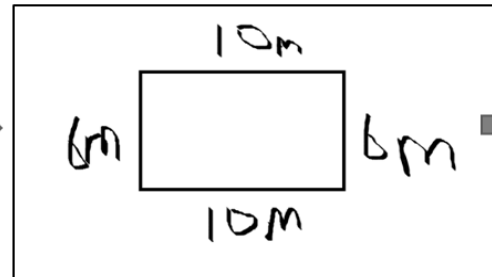
First I think we should make a table;

1 x 60
2 x 30
3 x 20
4 x 15
5 x 12
6 x 10

Then maybe from this we can figure out what the perimeter is. You'll have to take the two numbers away until you get a difference of 4 like e.g. 30 - 2 = 28. The answer is 6 x 10 and the difference between 6 and 10 is 4. Yay.

Answer: 10 + 10 + 6 + 6 = 32m²

This student has correctly solved the problem by creating a **table** and describing what she did in **words**.



She then drew a **diagram** using the **handwriting tool**.

$$\begin{array}{c} x+4 \\ \boxed{60m} \\ x \end{array}$$

$$x(x+4)$$

$$x^2 + 4x = 60$$

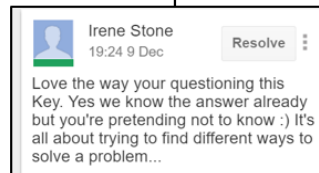
Another student starts the problem using **algebra**, again using the **handwriting tool**.

The student who originally solved the problem now wants to go back and see if she can solve it a different way. The teacher **comments** and encourages her to keep going.

$$(x+4)(x) = 60m^2$$

Well we know that x equals 6 but how do we find out that it is 6?

$$\begin{array}{c} x \quad 4 \\ \times \quad \times^2 \quad 4x \\ \hline x^2 + 4x \end{array}$$



$$\begin{array}{r} x^2 + 4x = 60 \\ -60 \\ \hline x^2 + 4x - 60 = 0 \\ (x + 10)(x - 6) = 0 \\ x = +6 \text{ or } -10 \\ x(\text{squared}) + 4x - 60 = 0 \end{array}$$

$$\begin{array}{r} 10 \quad -6 \\ \times \quad \times^2 \quad -6x \\ \hline 10x \quad -60 \end{array}$$

$$\begin{array}{r} 6 \quad -6 \\ \times \quad \times^2 \quad -6x \\ \hline 6x \quad -36 \end{array}$$

$$6(\text{squared}) + 4(6) = 60m$$

$$36 + 24 = 60$$

A student uses the **g(Math)** feature in Google Docs to insert an **algebraic equation**.

- It was the collaborative nature of the online environment that encouraged students to demonstrate different ways to solve a problem.

“Well one person in our group would maybe write a paragraph and explain it and then another person would go and do like in a table and then another one would do in a graph so it was kinda good to see all the different ways and if you got one way and still no one was on you'd try and find another way”

Scaffolding

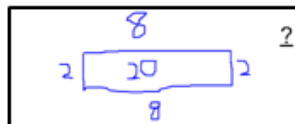
- .. by teacher and student

“Viewing each person's perspective like what they said and you can add on... add stuff to it... to make it better”

“I think X started a problem and I finished it”

“It was slightly confusing what to do on the 4th problem but then like X gave me a brief understanding of the problem so I managed to solve the problem on my own even having like you at home (the teacher) and it just felt more like we were learning more in-depth from the class”

- Allows for formative feedback (NCCA, n.d.). According to Hattie (1999), feedback is the most important thing a teacher can do to enhance student achievement; it's about helping students knowing how and why they understand or misunderstand, “what directions the student must take to improve” and “matching the next teaching act to the present understandings of the student”



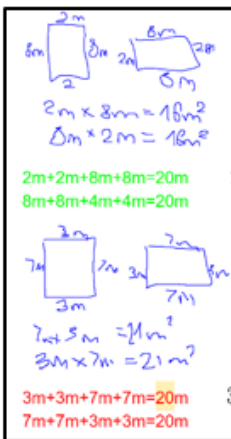
does everyone agree with me on this? This is the area of a 20m rectangle but I'm not too sure what the maximum rectangular means. Does it mean the largest numbers you can multiply together to make the area? Or am I totally **wrong**?

Irene Stone
18:02 6 Dec

you are not wrong. Area is when you multiply the numbers together... keep at it :)

Ok thank you

?: yes there are different way's try to look for more 4 numbers that add up to 20.
"Does it mean the largest numbers you can multiply together to make the area?"
 In a way you are right, try to look for the **largest area**, not numbers.



Irene Stone
19:17 6 Dec

a well done you are on the right track.. keep going!! it might help to make a table..

Student A does not understand the problem. The teacher tells her she is correct about area (when you multiply the numbers). However the teacher holds back from telling her everything and waits to see if another student responds.

Student B responds to student A. She recognises the misconception that student A had with finding largest numbers, not area. This is an example of peer-peer scaffolding. She doesn't tell her the answer but instead points out the misconception and points her in the right direction.

Student B continues to draw different rectangles. Examples of some of the rectangles she drew are shown. The teacher scaffolds her learning by giving a tip to create a table.

Student B creates the table. The teacher challenges her to draw a graph. Students have not drawn quadratic graphs before. This is tapping into the ZPD of the student.

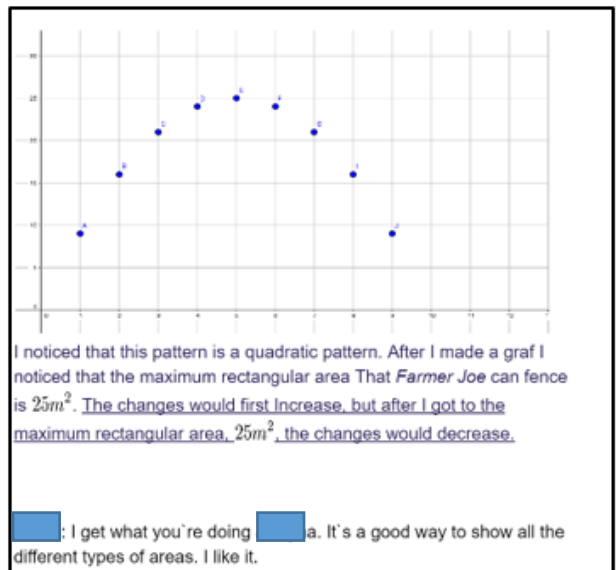
Student B figured out that the pattern was quadratic, something that wasn't required in this problem. This is clear evidence of further learning and would not have happened without intervention of more knowledgeable other – "the teacher" who asked her to draw a table. Student B draws the graph and explains what she has done. Student A comments underneath that she understands.

Lenth (m)	Width (m)	Total (m)	area (m ²)	changes:
1	9	20	9m ²	+7m
2	8	20	16m ²	+5m
3	7	20	21m ²	+3m
4	6	20	24m ²	+1m
5	5	20	25m ²	-3m
6	4	20	24m ²	-1m
7	3	20	21m ²	-3m
8	2	20	16m ²	-5m
9	1	20	9m ²	-7m

Irene Stone
19:53 6 Dec

This is amazing work girls. I'm very impressed well done!!! Can you see what type of pattern these numbers are? (i.e. the Area numbers).

This is a big challenge: would you be able to GRAPH the area numbers against the length? (Put the length on the X axis)



Collaborating

- Collaboration supports problem solving (Kim & Hannafin, 2011; Tanner & Jones, 2000)
- The Google Doc online environment allows students to communicate and help each other

“I think it was better for more independent learners that usually would study by themselves or write down everything in their copy, it was better for them to interact with other people and see that there is more than one way they do it”

- Collaboration is an important key skill; Junior Cycle, Digital Strategy, Project Maths curriculum

$$\frac{1}{4}(x) + 3x = 65$$

X all by 4

$$1(4x) + 12x = 260 =$$

$$4x + 12x = 260$$

$$16x = 260$$

$$\text{If } x = 16 \frac{1}{2}$$

$$\frac{1}{4}(16 \frac{1}{4})$$

11:38 1 Dec Resolve

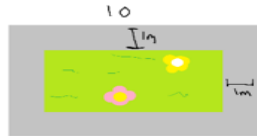
I think you mean that X= 4 on your second line? If so, then, your 3rd line should be $1(4) + 12$. You should get a completely different answer if you work it out

Irene Stone
14:06 1 Dec

This is amazing! well done to both of you. Fantastic you are trying to solve problem different ways and spotting mistakes in each other's work: true collaboration.. Keep at it :)

A student makes a mistake and another student helps her in the comments.

3) To do this one, we need to use our imaginations a bit.



Logically, because the grey area is 1m wide, and it's a 4 sided shape, so to figure out the dimensions for the green area, we must take 2m from the width and length. So 10 and 8 turns into 8 and six. $8 \times 6 = 48m^2$

Hi , this is regarding your answer for question 3. I understand that have take away from the length and width but why do we have to take away 2 instead of 1?

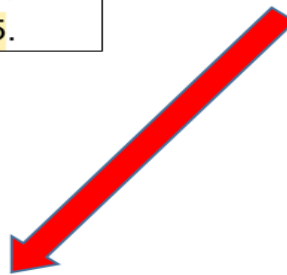
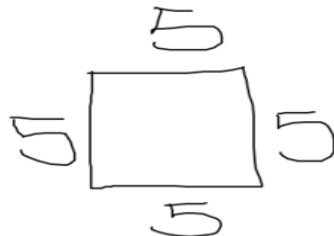
i think it's because there is 2 sides so you have to take 1m from both sides. Basically what you do on one side you have to do on the other.

Students helping each other in the Google Doc – they are typing in different colours.

Well, I was thinking that maybe 20m is the total perimeter of the rectangle so if you were to divide it by four (because a rectangle has 4 sides) you would get 5 BUT it's a rectangle so it couldn't be 5.

A student thinks that a square is not a rectangle. Another students addresses her misconception.

Ms said that a square is also a rectangle since a rectangle and square are shapes in which the opposite sides are equal so we can use 5×5 .



Motivation

- The collaborative online environment introduced a level of competition and raised the level of expectation of what an individual can achieve..

“It helped me ... to be able to get formulas for different harder problems”

- Students pushing themselves to learn new things. Student learning how to solve a problem using Logs (Leaving Cert standard)..

ii) Well, if we fill in the information in the formula, we'll get this:

$$13.98 = 15(0.994)^x$$

Anything underlined is the time, because I can't get it to look like a smaller square symbol.

So, there's no mathematical way to figure X out other than trial and error, using simple logic.

So, we know that $15(0.994)^3 = 14.73$

So, the times number has to be bigger than 3, because 13.98 is smaller than 14.73.

So, my first guess was 9. When I got 14.2091702748, I knew I had to go bigger. I then guessed 12, and when I got 13.9549367311, I had passed it, but not by much. I immediately checked 11, but that was 14.0391717616, so it was then I realised it had to be a decimal number, and a high one at that. So, I checked 11.5, 11.6, until I eventually found that $15(0.994)^{11.7} = 13.9801540337$, which I rounded to 13.98.

So really, it's all trial and error. I'm not the best at explaining, but I still hope I helped :)

Irene Stone
09:01 21 Jan

There is another way you can solve this problem, however it is something from the Leaving Cert Maths course. So not needed at this stage. Basically you are looking for a function that is going backwards of indices. So, opposite of multiplication is division isn't it? So what is the opposite of "to the power of"? I can show you if you like or you could try and find out for yourself??? :) (Google search maybe?)

Show less

Irene Stone
09:02 21 Jan

<https://www.mathsisfun.com/algebra/logarithms.html>

Irene Stone
11:39 22 Jan

look at what I did below....

Irene Stone
08:59 21 Jan

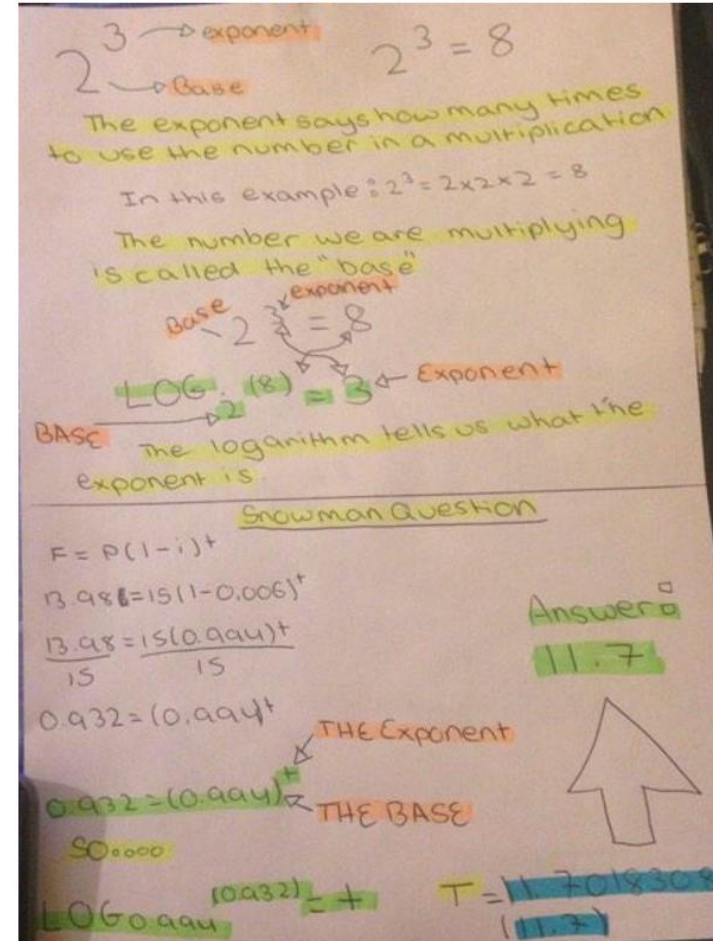
Well done

This is great :)

I only answered (ii) i am not sure if this is right, i looked at the website and tried to do it.

The top part of the page is just an example.

The bottom part is the question.



Irene Stone
11:38 22 Jan

this is AMAZING well done :)

This is Leaving Cert higher level!!! You obviously are not expected to know this for Junior Cert but no harm.. this is great. Do you see how you got the same answer as I? Well done :) I'm very impressed.

Student A: Trial and Error method

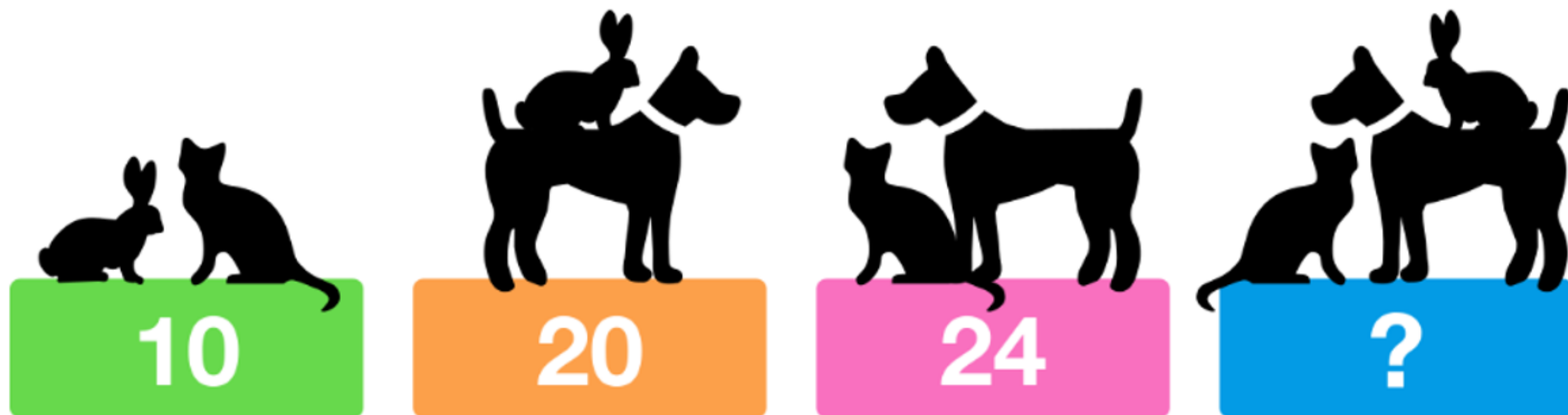
Student B: Using Logs method

Whole school approach to embedding problem solving

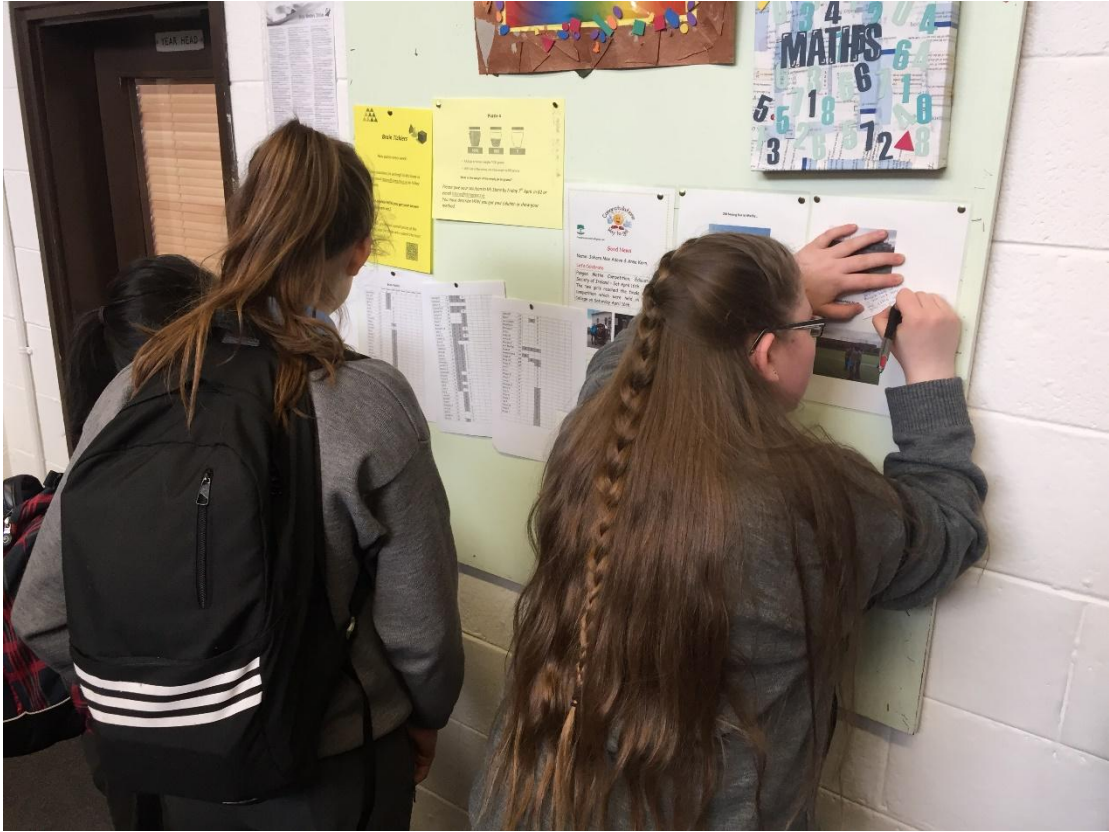
- Numeracy initiative
- Weekly puzzle competition for all students in the school
- New puzzle hung up every week
- Students submit their solutions to me on paper or by email
- Students have to describe how they get their answers
- The word “maths” not used
- No pressure put on students or teachers
- Teachers (non Maths) doing it with their students during tutor time
- Weekly prizes
- Prizes will be awarded at the end of the year

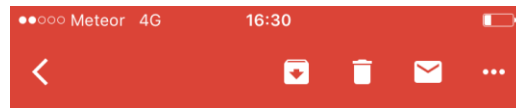
Puzzle 6

What weight will the fourth scale display?



Think carefully! There's a quick and clever shortcut for solving this.





This week's Brian Tickler Question Inbox ☆



to me

16:25 ...

Ms. on the sheet you printed on the last part [redacted] and I couldn't tell if there was 4 or 3 Bananas on the last part of the question when counted and we just wanted to verify with you because we already submitted our answers because Mr.McGarry collected them and we only relised afterwards. (We got 15 but we need to know because if the image was 3 we would need to change the answers on our submissions to 14)

I forgot to write on the answer I gave you that its 2D because $20 + 24 - 10 = 34$ (2 Dogs) I don't know why I said substitute and I guess it still works but I was looking at it confused and found a way to summarize it:

$$20 (\text{rabbit \& dog}) + 24 (\text{cat \& dog}) - 10 (\text{cat \& rabbit}) = 34 (2 \text{ dogs})$$

$$34 / 2 = 17$$

$$\text{Dog's weight} = 17$$

$$17 + 10 = 27 (\text{dog} + \text{rabbit} + \text{cat})$$

The - 10 Balances it out to 2 dogs (34)

Sorry about that I was going through it with someone I meet at The Walton Club and he said I did it weird so I went through it again the other way shows all the animals individual weights though so this is more of an add on then full answer again sorry about that.

Hi Ms Stone

I looked at the sheet today and saw that I got the Question wrong. I asked [redacted] which answer she got and she got also 30. I gave my sheet on the Friday before the exams started. I had to ways of answering it (one with my guess and one the algebraic way) I just want to check if I got it right or wrong because I was really interested in that Question.

-I hope I caused no inconvenience but it was just out of curiosity -

Thanks,

[redacted] 😊

EAL teacher

- *“Today the language support programme was visited by two inspectors... they were in A4 for about an hour to chat to some of the students, see how the programme works, look at projects etc... anyway, at one stage they looked around and realised that about 8 students (some first and second years, a fifth year) in different corners of the room were all working on this week's puzzle, some with pen and paper, some using mini white boards, a couple on laptops. No one had told them to do it... it was just that when two students took the puzzle down from the whiteboard and started on it, and then others saw them working on it, they wanted to too..... They were blown away - they said how great it was to see students working so enthusiastically on maths... and especially because they had chosen to do it, not been told to.”*

3 apples = 30 so we divide
30 by 3 = 10 so each apple
= 10

$$\text{Apple} = 10 + \text{Bananas} + \text{Bananas} = 18$$

Since we know an apple = 10
 $18 - 10 = 8 \div 2$ (Pairs of bananas) = 4

Bananas = 4 which happens to be the
amount in the image.

$$4 \text{ Bananas} - 2 \text{ half coconuts} = 2$$

Since we know from the last question
1 the image Bananas = 4 so

$$4 - 2 = 2 = 2 \div 2 = 1 \text{ half coconut}$$

$$\text{Apple} = 10$$

Banana = Amount in image (4 in last question)

$$\text{Half coconut} = 1$$

Half coconut + Apple + Pair of 3 bananas

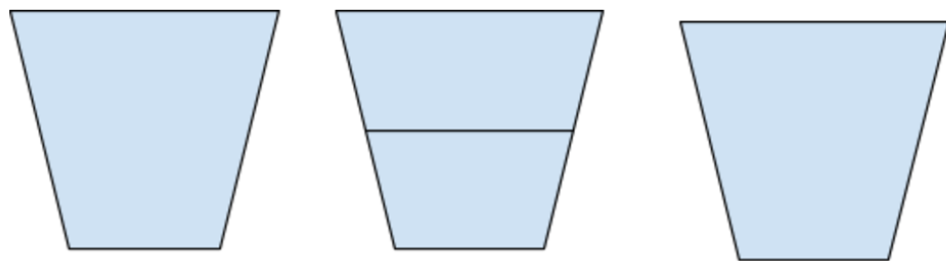
Bananas = 3 because when compared
to other image there is 1
less banana.

$$\text{So... } 1 + 10 + 3 = \underline{\underline{14}}$$

$$\begin{array}{c} (10) \quad (10) \quad (10) \\ \text{Apple} + \text{Apple} + \text{Apple} = 30 \\ (10) \quad (4) \quad (4) \\ \text{Apple} + \text{Banana} + \text{Banana} = 18 \end{array}$$

$$(4) \text{ Banana} - (2) \text{ Half Coconut} = 2$$

$$\begin{array}{c} (10) \quad (3) \quad (1) \\ \text{Apple} + \text{Banana} + \text{Half Coconut} = 14 \end{array}$$



1000g

600g

???

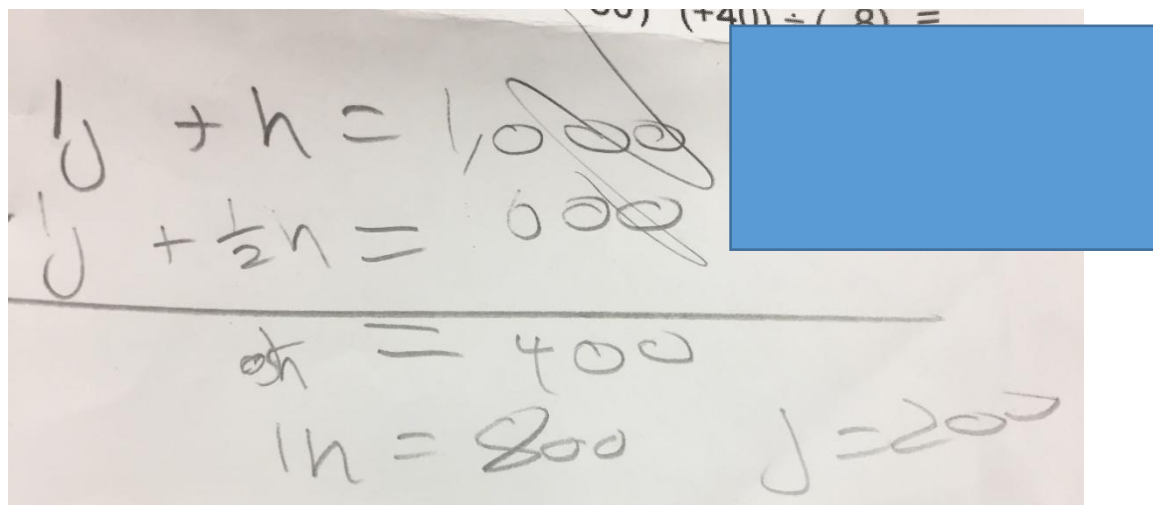
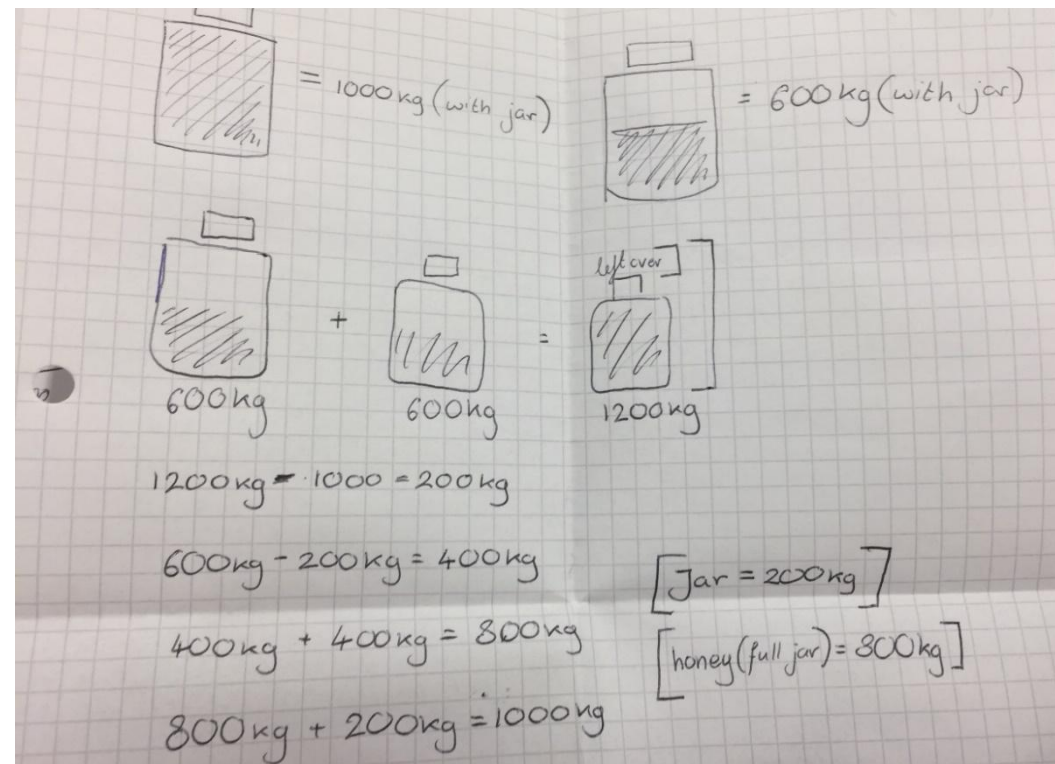
$$600 + ? = 1000$$

$$600 + 400 = 1000$$

$$400 + 400 = 800$$

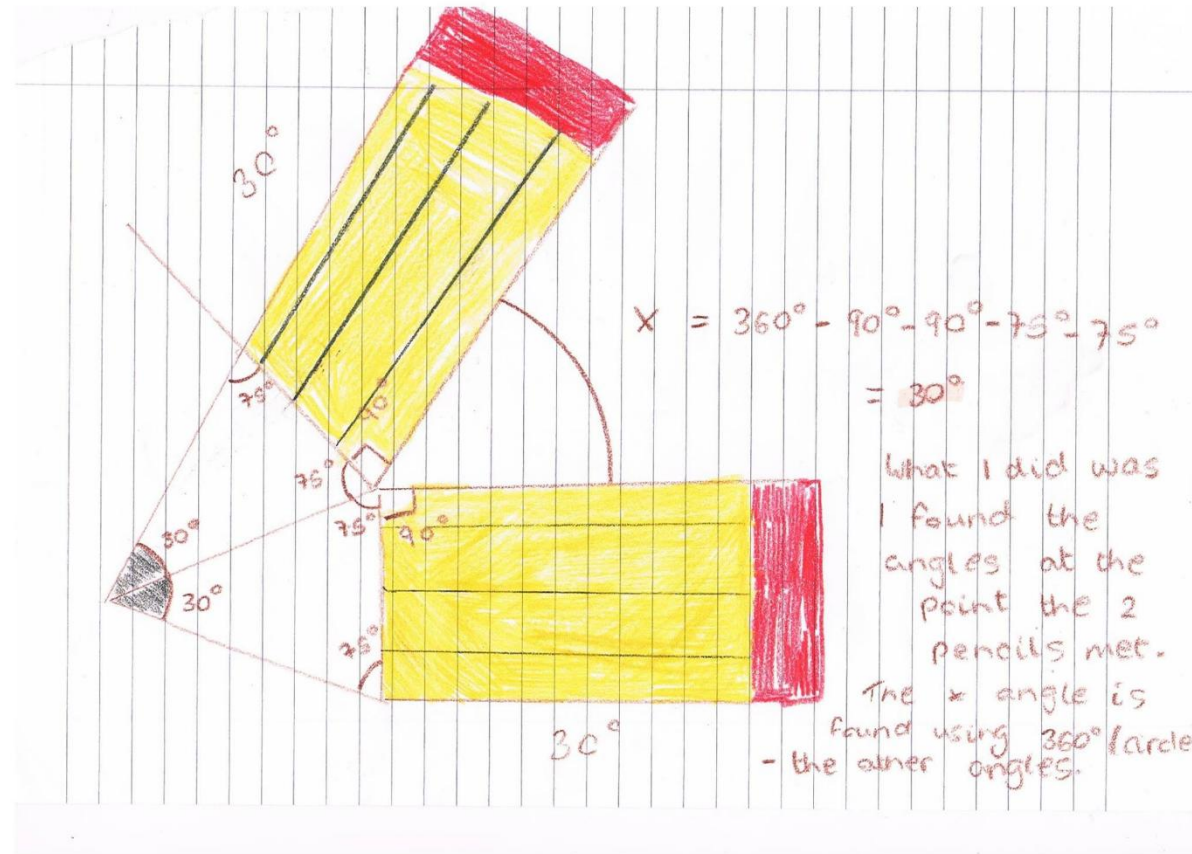
So $1000 - 800 = 200g$

The empty jar weighs 200g!



The answer is 200g because if a full jar of honey holds 1000g and half a jar holds 600g you have to take a amount away that will go into bought numbers evenly. A = 200g

Creativity



- *“I entered the competition because when I looked at the first puzzle, I was immediately interested in it... What I like about the puzzle competition is that first it looks easy but actually it isn't always as easy as it looks!(fruit puzzle)”*

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